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Contents

Zermelo's Philosophy of Mathematics

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Keywords: Zermelo, set theory, axiomatization, philosophy of mathematics

Abstract

The most well-known and arguably the most important mathematical contribution of Ernst Zermelo is his axiomatization of set theory. The introduction of axiomatization opened many possibilities for further investigation of set theory, but also raised many philosophical questions. Unlike many mathematicians working in foundations in his time, Zermelo was not interested in philosophical aspects of his contributions. However, through careful analysis of his mathematical work, it is possible to reconstruct a significant portion of his philosophy of mathematics.

In this talk, we will present one interpretation of Zermelo's philosophical views about logic, set theory, and mathematics. We will argue that Zermelo's philosophy of mathematics, especially his views on axioms, is traditional in the sense that it does not incorporate Hilbert's formalistic approach.

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Mathematical–Logical Approach to Structuring Anti-Money Laundering Risk Models: Conditional Logic in Transaction Monitoring System

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Keywords: AML risk assessment models, transaction monitoring, mathematical logic, conditional logic, risk-based approach, risk assessment, formal methods

Abstract

Anti-Money Laundering risk assessment systems in modern financial institutions increasingly rely on quantitative models whose functioning depends on logical rules, assumptions, and structured decision mechanisms. According to regulatory guidance, such models are treated as formal quantitative methods and therefore require clearly defined assumptions and independent validation within a risk-based approach. This paper provides a methodological and logical analysis of a selected fragment of an AML risk assessment model, focusing on the transaction monitoring component, where detection rules are frequently formulated as logical conditional structures of the form "if ... then ...". The paper develops a formal representation of such conditions (logical formulas) and proposes a methodological framework for integrating automated rules with human judgement in the context of supervisory expectations and risk assessment processes. The analysis demonstrates how a mathematical-logical approach enhances transparency, auditability, and robustness of detection mechanisms, while clearly outlining model limitations and regulatory implications for risk-based AML frameworks.

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Syntactic Purity in Sequent Systems

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Keywords: syntactic purity, semantic pollution, sequent calculi, proof theory, philosophy of logic.

Abstract

Formal logic relies on proof systems for establishing the validity of formulas. However, a tension persists regarding the nature of these systems, namely the debate between syntactic purity and semantic pollution [7, 8, 3, 4, 13]. Traditionally, a proof system is considered syntactically pure if it operates without incorporating elements from semantics, such as possible worlds or truth values. Conversely, systems that explicitly embed such notions are deemed semantically polluted. So far, purity has been defined either negatively, as the mere absence of semantic elements [11], or through overly strict requirements for the total independence of syntax from semantics [1]. We address the need for a precise, positive characterization of syntactic purity within sequent calculi.

We propose a theoretical principle for syntactic purity in sequent calculi which state that a sequent calculus is syntactically pure if and only if its sequents admit a translation into a logical formula. This principle is independent of the particular logic under consideration, but regards only the calculus in itself. Based on this principle, we formulate a concrete syntactic purity criterion for standard sequent calculi, demonstrating that they qualify as syntactically pure. Moreover, we show that several generalizations of standard sequent systems (such as hypersequents, nested sequents, n-sided sequents, and display calculi [1, 2, 14, 6]) also satisfy our principle, fulfilling an opportune syntactic purity criterion for them. In contrast, we argue that labelled sequent calculi [9] constitute a form of semantic pollution. Because they employ explicit semantic elements derived from model-theoretic semantics, such as world labels and accessibility relations, which cannot be translated into a conditional logical formula, they fail to meet the proposed purity criterion.

A potential objection arises with First Degree Entailment (FDE) logic [10]. Due to the fact that FDE has no theorems, it may appear impossible to give a formula-interpretation of FDE sequents, which would threaten the claim that FDE sequent calculi [12, 5] are syntactically pure. A closer analysis, however, shows that this difficulty is only apparent and that the objection dissolves under the proposed account.

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Weak Impredicativity and *Grundgesetze*

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Keywords: impredicativity, Russell's paradox, Frege, Grundgesetze der Arithmetik

Abstract

The debate on impredicativity shows that this term can refer to different phenomena ([11]). In this talk, I will examine various forms of what Gödel termed "weak impredicativities" ([6]) and consider whether they could inform a consistent revision of Frege's logicist project ([15]).

At least three notions of predicativity/impredicativity are usually distinguished. These are "strict" predicativity, "relative or classical" predicativity, and "generalised or constructive" predicativity. Firstly, "strict" predicativity ([7]) is a syntactic notion captured by the following characterisation: a definition is said to be impredicative if its *definiens* contains a quantifier that binds a variable of the same order as the *definiendum*; in this case, the *definiendum* is one of the possible substitutional instances of the quantified *definiens* and apparently blocks the evaluation of the latter.[1] This notion of predicativity is the most austere, requiring the rejection of large portions of mathematics. In particular, with respect to this criterion, the induction scheme is impredicative; therefore, only fragments of arithmetic can be admitted ([10]). Secondly, "classical" predicativity ([10], [13] and [21]) is also known as "predicativity relative to natural numbers". This concept arose from Feferman and Kreisel's parallel investigation into the limits of predicativity, i.e. examining the extent to which predicative theories could be developed once natural numbers were accepted as given (despite their strictly impredicative definition). This interpretation of predicativity appears to be more promising, at least from a mathematical point of view. In this approach, natural numbers are assumed to be given, and a ramified hierarchy of second-order arithmetical theories with ramified comprehension is provided. Each stage is represented by a theory indexed by an ordinal available in the previous stage, and the limit of predicativity is identified as the first ordinal that cannot be proved predicatively (which has been proven to be Γ_0). Thirdly, "generalised" ([4], [18], [22]) or "constructive" predicativity ([5], [3], [1]) is instead a more recent criterion, introduced in order to capture a different shade of predicativity as constructivity. In virtue of this criterion we can classify as predicative also many inductive definitions.

With respect to this widely accepted taxonomy, I would argue that the notion of predicativity involved in strict and classical predicativism is originally

[1] A semantical description is maybe more usual. A definition is said to be "impredicative" if it contains an unrestricted quantifier that binds a variable of the same order of the *definiendum* – then, the *definiendum* is a substitutional instance of the bound variable.

syntactic. Conversely, the notion of generalised or constructive predicativity stems from an ontological characterisation of the structure of predicative entities. This characterisation is very similar to that captured by the set-theoretic criterion of well-foundedness (cf. [5], [3]). I support the idea that the weakly predicative phenomena that Gödel had in mind involve this ontological notion of predicativity, and I propose referring to the others as forms of "strong" impredicativity.

In view of the final application to Frege's system, a useful addition to the discussion on predicativity would be an investigation and comparison of the historical objections raised against impredicativity. These include paradoxicality ([7], [8]), vicious circularity ([9] and [19]), and instability ([20], [4], and [17]). We will substantiate the following claims. Firstly, paradoxicality implies vicious circularity, but not vice versa. This can easily be illustrated using Russell's example of non-paradoxical statements that are nevertheless viciously circular when used as definitions, such as "having all the properties of the great general" ([7]). Secondly, vicious circularity entails instability, but not vice versa. This claim is supported by examples of unstable classifications that are not viciously circular, such as "the younger men in the list" ([17]). Thirdly, instability presupposes impredicativity, but not vice versa. To exemplify this last claim, we will examine impredicative definitions in Frege's *Grundgesetze* and demonstrate that they maintain stability and are therefore reliable components of a logicist programme.

The second part of the talk is devoted to analysing second-order impredicativity in Frege's *Grundgesetze*. This investigation is guided by the idea that the predicative subsystems that have been proposed so far – [16], [23], [14], [2] – are obtained by applying "strongly" predicative restrictions, which in this context mean "strict" predicativity.[2] I will briefly discuss these results to demonstrate the interesting parallels with those of strict predicativism, as mentioned above. Strongly predicative restrictions of *Grundgesetze* can indeed derive a fragment of second-order Peano arithmetic (PA^2) equivalent to Robinson arithmetic (Q). In this case, too, predicativism requires us to renounce induction, even though it is not assumed as an axiom, but rather derived as a theorem. Furthermore, the derivation of arithmetic does not proceed, not even partially, via Frege's definitions, as most of the original definitions, particularly the crucial notion of "ancestral" (cf. note 3), are precluded. Therefore, even with respect to the Neologicist proposal (cf. [12]), a predicativist restriction would undermine the derivation of Frege's theorem (FT), i.e. the derivation of second-order Peano axioms from second-order logic augmented with Hume's principle and Frege's definitions (ancestral, weak ancestral, predecessor and natural number).[3] In other words, strong predicativism excludes a feature that appears to be involved in the derivation of Russell's paradox, but was also necessary for the logicist derivation of Frege's theorem.

Conversely, I emphasise that the effect of a "weakly predicative" approach has so far been unexplored, and I aim to demonstrate that it would introduce an

[2]The classical approach of taking natural numbers for granted would not be a good starting point for the logicist programme of deriving arithmetic from a logical system augmented with some definitions.

[3]Ancestral of R: R*(xy) $\equiv_{def} \forall X((\forall z(Rxz \to Xz)) \wedge (Her(X,R) \to Xy))$Weak Ancestral of R: $R^+(x,y) \equiv_{def} R^*(x,y) \vee x = y$. Predecessor: P(x,y) $\equiv_{def} \exists X \exists z(Xz \wedge y = \sharp X \wedge x = \sharp[Xw \wedge w \neq z])$. Natural Number: $\mathbb{N}x \equiv_{def} P^+(0,x)$.

interesting modification to Frege's *Grundgesetze* because it would prevent the specification of Russell's concept ($Rx \leftrightarrow \exists X(x = \epsilon X \wedge \neg Xx)$), yet remain open to Frege's vocabulary particularly the definitions of ancestral, weak ancestral, predecessor and natural number (cf. note 3). Furthermore, it would enable the derivation of second-order Peano axioms, including induction.

In the final part of the talk, we will examine a potentially weaker form of impredicativity that remains compatible with both Gödel suggestion and Frege's logicist purposes. More precisely, we will identify and test a consistent and powerful predicative restriction of the comprehension axiom schema: $\forall \alpha R(D, \alpha) \leftrightarrow \forall y(\phi(y) \rightarrow R(\alpha, y))$ – where R is the characteristic relation for elements of the type of the *definiendum* (D), i.e. the relation used to provide its identity conditions. We will demonstrate that such a scheme is compatible with the Fregean vocabulary and the weakest form of impredicativity discussed so far. This will enable us to derive an appropriate version of the Fregean logicist project that remains faithful to the *Grundgestze* proposal.

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Teaching computers set theory

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Keywords: set theory, New Foundations, type theory, computer-assisted mathematics

Abstract

Nowadays, in the age of AI, we use computers more and more like partners, and less like tools they used to be. Some of these uses encompass doing mathematics. We had proof assistants even before the LLM craze of the last few years. However, there's a problem. The usual paradigm under which the theorems are proved by humans is based on set theory (usually ZLC), but those proofs are very inconvenient for computers to work with (both to find and to check).

Proofs done by human–computer cooperation usually use some variant of type theory, in order to enable the computer to keep track of what's possible and sensible to do with which objects (in ZFC it is famously possible to produce abominations such as $\bigcup \pi \times \mathcal{P}(7)$, and only conventions dictate which operations are sensible in which contexts). However, type theory has its own drawbacks, mostly in added layers of bureaucracy, and impossibility of expressing natively certain concepts such as subsets.

NFU (New Foundations with Urelements) provides an interesting compromise: with a minimum of types for computers to orient themselves, yet similar enough to ZFC that most results from there can be transferred without much effort. We will present a prototype of a software system for expanding the language of set theory, with automatic type inference and detection of untypable formulas.

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Is reality logical? Nietzsche *contra* Aristotle

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Keywords: principle of non-contradiction, Aristotle, Nietzsche, logic, metaphysics.

Abstract

One of the earliest formulations of the principle of non-contradiction can be found in Aristotle's *Metaphysics* (Γ 3, 1006a), where he states that "the same thing cannot at the same time belong and also not belong to the same thing and in the same respect." For Aristotle, the principle of non-contradiction (PNC) is the firmest of all principles, and it is not something that can be demonstrated, since every demonstration already presupposes it. For him, the PNC is not only a fundamental axiom of reasoning, but also a principle that mirrors the structure of being and reality.

On the other hand, Nietzsche, who is committed to a broadly Heraclitean conception of reality as flux and becoming—a doctrine explicitly rejected by Aristotle—can appear to be a thinker who rejects the PNC altogether. The purpose of this talk is to argue that Nietzsche only rejects the ontological version of the PNC as found in Aristotle, not its qualified logical version. For Nietzsche, logical principles are not discovered as features of a mind-independent reality but are created as tools through which human beings render the world "expressible and calculable" (WP, §516). Accordingly, whoever claims that there are no contradictions in reality, can only do so because reality has already been made logical through conceptualization. However, this does not mean that Nietzsche objects to the application of logical principles to propositions within human discourse. On the contrary, he himself invokes the principle of non-contradiction when rejecting Kant's thing in itself (BGE §16) and the concept of a *causa sui* as self-contradictory (BGE §21). Overall, this paper clarifies both Nietzsche's attitude toward logic and the metaphysical presuppositions of Aristotelian logic.

Part 1 of my presentation provides an overview of Aristotle's logical and ontological formulations of the PNC. Part 2 presents Nietzsche's ideas on logic and the PNC, arguing that Nietzsche rejects only the ontological version of the principle. Part 3 argues that Nietzsche's and Aristotle's fundamental disagreement rests on the question of whether reality is inherently logical or is made logical by our conceptualization.

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Advantages of the new sequent system for IL

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Keywords: interpretability logic, sequent system, cut elimination, uniform interpolation

Abstract

Interpretability logic [6] extends provability logic, which is a modal treatment of Gödel's provability predicate. Its language has an additional binary modality, which corresponds to the notion of relative interpretability between first-order arithmetical theories.

Although there are many papers on various semantics for interpretability logics, there are not many papers on sequent systems for interpretability logics. Sequent systems for interpretability logic IL and its extension ILP were introduced by K. Sasaki [5]. Sequent systems for some sublogics of IL were studied by the group of S. Iwata, T. Kurahashi and Y. Okawa [3]. Using the non-wellfounded proof theory [4], a theory that allows proofs to have infinite height, we have introduced the non-wellfounded sequent system $\mathrm{G}^{\infty}\mathrm{IL}$ for the interpretability logic IL in [1].

In this talk, we will analyze the above mentioned sequent systems for the interpretability logic IL and its extensions. We will present the advantages of the new sequent system, i.e. we will list the results obtained using the new system that are new or obtained in a easier way than before. These results include cut elimination [1] and uniform interpolation [2].

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Computable approximations of semicomputable continua

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Keywords: computable set, semicomputable set, continuum

Abstract

A semicomputable set in Euclidean space need not be computable. However, under certain conditions it is possible to approximate a semicomputable set by its computable subset (in the sense of Hausdorff metric). In particular, there are conditions under which a semicomputable continuum K (i.e. a semicomputable set which is compact and connected) can be approximated by its computable subcontinuum.

In this talk we discuss these conditions and we are focused on the case when K is a chainable continuum.

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New topological semantics for interpretability logic

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Keywords: interpretability logic, topological semantics, Veltman semantics

Abstract

Interpretability logic **IL**, introduced by Visser [3], is an extension of provability logic **GL** with binary modal operator $\rhd$. One of the most well-known semantics for **IL** is the Veltman semantics for which **IL** is complete, but not strongly complete.

Iwata and Kurahashi [2] introduced topological semantics for **IL** that corresponds to lesser known Visser semantics, sometimes also called simplified Veltman semantics. They also proved topological strong completeness of **IL** and some of its extensions.

In this talk, we introduce new topological semantics for **IL** that corresponds to Veltman semantics and discuss completeness and finite model property of **IL** and its extensions for this semantics.

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Noise, Novelty, and (Non)computability

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Keywords: noisy images, structural extraction, Rényi entropy, stability map, algorithmic information theory, Kolmogorov complexity, computable proxies, novelty and typicality, Mandelbrot-like sets

Abstract

We present EntropyEye, a dual-branch consensus algorithm for extracting stable structure from noisy digital images, motivated by digital art restoration and forensic material analysis. The image is processed in parallel by an intensity branch and a local Rényi-entropy branch; we then slice the dynamic range at multiple threshold levels. At each slice, we apply a threshold rule to select, adaptively, a cut between background-dominated and feature-bearing clusters. A pixel is accepted only when supported by both branches, and decisions are accumulated across slices to yield a graded stability map. The design is inspired by our earlier "overlapping supports" approach for extracting useful information from noisy time–frequency representations [1] and its extension to time-frequency-inspired structural image analysis [2].

We use EntropyEye as a case study in computable proxies for logic-familiar distinctions between randomness and structure. Kolmogorov complexity formalizes randomness as incompressibility but is noncomputable; practical analysis therefore relies on computable surrogates and model-based notions such as typicality and novelty [4]. We relate EntropyEye's retention of noise spikes and rejection of persistent textured structure to AIT-motivated frameworks for computational creativity and aesthetics [3]. For controlled experiments, we treat finite renderings of Mandelbrot-like sets as reproducible multiscale calibration objects, while emphasizing computability and complexity caveats for the underlying dynamical loci near boundaries [5, 6, 7]. Finally, we discuss style-dependent behavior on paintings, including cases where impressionist brushstroke fields are mapped as coherent structure rather than filtered as noise.

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Gentzen's Sequent Calculus and Beth Tableaux

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Keywords: Gentzen sequent calculus, Beth tableaux

Abstract

The paper presents an introduction to Gentzen's sequent calculus and Beth tableaux as two fundamental methods of proof in classical propositional logic. It outlines the historical background and motivations behind Gentzen's approach, emphasizing the elimination of non-essential formulas from proofs and the subformula property, which leads to decidability results. The formal language of the sequent calculus, together with its structural and logical rules, is described. In parallel, Beth tableaux are introduced as a semantic method based on systematic truth assignments. Several classical logical laws, including contraposition, Clavius' law, and hypothetical syllogism, are demonstrated using both Gentzen proofs and Beth tableaux, highlighting the relationship and complementarity between syntactic and semantic proof techniques.

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A formal system of dialectics

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Keywords: dialectics, dialectical logic, logic, modal logic, interpretability logic, frame semantics

Abstract

Dialectical reasoning, placing emphasis on contradictions and their resolutions, has been studied since the ancient times. It describes a dynamic mode of development, describing processes of transformation rather than static states. It was included as a core principle in a variety of different philosophical schools of thought by figures such as Plato [1], Aristotle [7], Hegel [2], and Marx [6]. Despite this, dialectical reasoning lacks a clear logical articulation.

Even though some attempts were made using paraconsistent logics (see, e.g., [8]) or category theory (see, e.g., [3], [4] or [5]), in this talk we attempt to provide a formal system of dialectics, **DiaL**, as an extension of modal logic **K**. Dialectical contradiction is here considered as a logical connective (distinct from logical contradiction), which is included in the language and the structure of our reasoning, as opposed to it being something defined within a bigger theory. System **DiaL**, similarly to **K** in modal logic, plays a role of a base system for a class of logics, i.e., systems of interest are constructed from it by adding additional axioms. We call such systems dialectical logics.

We note that system **DiaL** is a sublogic of system **IL** (which is a base system of interpretability logics, see, e.g. [9], [10], [11]), up to a certain translation. This provides us with a simple way to define models for dialectical logics, as Kripke frames with some additional structure.

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On Newcomb's problem

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Keywords: Newcomb's problem

Abstract

We explain in a novel way why the *two boxes argument* is a rational solution to Newcomb's problem and *one box argument* is not. If someone is confused by the fact that it is more profitable to be irrational than to be rational, we refer to the fact that there are many games where it is more profitable to be irrational and we give some examples.

Furthermore, we address the question: How comes that there are rational one-boxers? We offer a qualitative and quantitative explanation. Finally we prove that Newcomb's problem is not a version of Prisoner's dilemma, as sometimes asserted.

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Categorical logic in difference and differential algebra

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Abstract

We present a novel approach to algebra and geometry of rings and modules with operators by using the methods of categorical logic and category theory.

We view difference algebra as algebra internal to the topos of difference sets, and difference algebraic geometry as relative algebraic geometry. This allows for rapid progress in the study of homological algebra of difference rings and modules and cohomology theories of difference schemes.

By viewing differential rings and schemes as precategory actions, we develop the theory of descent in differential algebraic geometry, and obtain a major generalisation of differential Galois theory to the context of differential schemes.

We will argue that these developments are instances of a more general principle that has a significant potential for further applications.

“Bad grammar as well as bad logic”: Christine Ladd-Franklin on Bertrand Russell and his logic

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Keywords: Bertrand Russell, Christine Ladd-Franklin, logic

Abstract

Christine Ladd-Franklin (1847–1930), a student of C.S. Peirce, is recognized as America’s first woman logician. She wrote widely on algebras of logic; implication and existence; and the philosophy of logic, with special regard to which of the logical developments at the turn of the 20th century were best suited for making sense of philosophical and every day reasoning. Scholars of Bertrand Russell, if they are familiar with Christine Ladd-Franklin, know her because of a dismissive anecdote concerning her that Russell relates in [4, p. 196]. But this single anecdote does not capture the complexity of the relationship between the two, as it entirely erases Ladd-Franklin’s critical approach to Russell’s logic. The criticisms she levels against Russell’s logic have receive little to no attention in the literature (either on Russell or on Ladd-Franklin), in part because the most significant of her criticisms currently exist only in unpublished notes in her archives. The aim of this paper is to present Ladd-Franklin’s critical account of Russell and his logic, drawing on both her published and manuscript material [1, 2, 3]. This allows us not only to understand Ladd-Franklin’s own views on logic and philosophy of logic better, but also to properly reposition her with respect to Russell: Far from being merely the source of an amusing anecdote, Ladd-Franklin provides insightful criticism of his approach to logic which deserves serious consideration not only by historians of logic but also by contemporary philosophers.

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Relativizing Cardinality: Persistent Simultaneous Marking in $M \models$ ZFC and Layered Diagonal Construction in Wang's Σ-model

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Keywords: Set-theoretic foundations, Continuum Hypothesis, Löwenheim–Skolem phenomenon, model relativity, Dedekind completeness, diagonalization, Wang's Σ-model

Abstract

A central lesson of modern set theory is that size is not absolute: whether a set is *countable*, and what the cardinality of the continuum is, depends on the surrounding universe in which these claims are interpreted. The Löwenheim–Skolem phenomenon, the independence of the Continuum Hypothesis (CH), and multiverse perspectives make this model-dependence explicit. Motivated by this relativity of cardinality, and by Gödel's general idea of building richer domains from simpler ones via structured, definable expansions, we study two complementary constructions that formalize the separation between external size and internal completeness.

(1) Model-relative persistence in $M \models$ ZFC. In an ambient metatheory V, fix a structure $M \models$ ZFC. Working inside M, let S_Q^M be the family of closed rational-endpoint intervals and $S_{Q,+}^M$ its proper subfamily. We define a marking function $x : S_{Q,+}^M \to (\mathbb{R}^M \setminus \mathbb{Q}^M)$ with $x(I) \in I$, and the induced set $S_m^M(x) := \mathbb{Q}^M \cup \operatorname{rng}(x)$. We impose a chain-level coherence requirement, *persistent simultaneous marking*: for every nested rational-endpoint ω-sequence $(J_n)_{n\in\omega}$ in M with $\bigcap_{n\in\omega} J_n = \{r\}$, the collapse point satisfies $r \in S_m^M(x)$ (equivalently, $\bigcap_{n\in\omega}(J_n \cap S_m^M(x)) \neq \varnothing$). Under this persistence schema we prove, internally to M, and expressed as satisfaction statements $M \models (\cdot)$, that $S_m^M(x)$ is dense in $\mathbb{R}^M$, sequentially closed (hence closed), complete in the inherited metric, and Dedekind complete (every nonempty $A \subseteq S_m^M(x)$ bounded above in $\mathbb{R}^M$ has $\sup_{\mathbb{R}^M}(A) \in S_m^M(x)$). Expressing the conclusions as $M \models (\cdot)$ highlights the internal/external distinction familiar from the Löwenheim–Skolem phenomenon.

(2) Layered diagonal generation in Wang's Σ-model. We also develop an explicit layered diagonalization in Wang's Σ-model (a framework distinct from ZFC) following its hierarchical architecture. Starting from the 0-th layer Σ_0 of

rationals (with a fixed enumeration), we perform ω-many diagonal procedures to generate explicitly defined irrationals and adjoin them to obtain Σ_1. Iterating stage-by-stage, each Σ_{n+1} is obtained from Σ_n by ω diagonalizations, and the process continues through ω layers, yielding a constructively presented "atomic" representation of the continuum in the Σ-framework. This yields what we believe is the first fully explicit instance of Wang's Σ-construction.

We conclude by comparing these perspectives with standard ZFC practice, emphasizing what model-relative persistence in $M \models$ ZFC and layered diagonal generation in Wang's Σ-model each clarifies about definability, internal completeness, and the relativity of cardinality.

Rescuing Quantum Logic from a Category Mistake: On the Historical Heterogeneity of Logic

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Keywords: quantum logic, algebraic logic, inferentialism, history of logic, logical pluralism

Abstract

The logic von Neumann first proposed [13] and later developed with Birkhoff [4]—now known as BvN quantum logic—is often deemed a failure, typically due to its lack of distributivity and implication. Yet such criticisms, from Dummett's inferential realism [6] to Abramsky and Coecke's dismissal of BvN as "non-logic" [2], rest on a category mistake. They evaluate an algebraic–Boolean structure using criteria drawn from a deductive–Fregean conception of logic that was normalised only with the post-Gödelian unification of logic following his incompleteness theorems [9], which subsumed two distinct 19th-century traditions—algebraic and inferential—into a single field [11], erasing earlier conceptual heterogeneity.

This paper reconstructs that split: Boole's algebraic logic, centered on structural relations among propositions, departing from the syllogistic tradition [5], and Frege's inferential logic emphasizing judgment, as visually expressed in the two-dimensional layout of his "concept-script" [7]. I argue BvN logic belongs to the former, but not the latter. It was never intended to include implication in the Fregean sense; to fault it for this is to misclassify its kind.

Putnam's defense of quantum logic [12], drawing an analogy with the revision of geometry in relativity, rightly interpreted BvN logic as an internal structure of quantum mechanics, but wrongly extended it to replace classical logic altogether. Dummett's critique, though correct, only noted it cannot serve as a general inferential system or meta-language. Their apparent disagreement dissolves once we distinguish logic as internal structure, deductive calculus, and meta-language. Subsequent attempts to replace it with linear logic [8] or categorical semantics [1] repeat the same confusion. The marginalisation of quantum logic stems not from internal flaws, but from a historical forgetting of what logic once was.

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